Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure or vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Symmetrical Singularity Model for Lifting Potential Flow Analysis

Brian Maskew* and Frank A. Woodward†
Analytical Methods, Inc., Bellevue, Wash.

Introduction

THE method developed by Rubbert and Saaris et al.[1] for three-dimensional lifting potential flow problems has received wide use as a practical aerodynamic analysis tool. An improved method was later developed by Hess,[2] who provided circulation around lifting elements by adding a surface dipole (vorticity) sheet in place of the interior vortex lattice system used in Ref. 1. Both methods have a practical difficulty in applying the Kutta trailing edge condition (discussed in Ref. 2), as a result of which, significant variations can be obtained in surface pressures near the trailing edge and in overall lift coefficient.

Recent experience in applying these methods to slotted flap configurations emphasizes the difficulty in obtaining the correct pressure distribution, particularly in the slot region, and also the correct overall circulation. The purpose of this paper is to present some thoughts on the reasons for these problems, and to propose a possible solution in the form of a modified singularity model for the basic aerodynamic representation. Some calculated results are presented from a simple two-dimensional method incorporating the new model.

Basic Considerations

In the methods of Refs. 1 and 2, the surface source strengths are solved in the presence of a prescribed vorticity distribution. Figure 1 shows the source distribution obtained using a base method utilizing a uniform chordwise vorticity distribution similar to Ref. 2. Because the assumed vorticity distribution does not represent the correct loading distribution, the source distribution provides the necessary adjustment (doublet effect) by forming source-sink pairs between the upper and lower surfaces. The magnitude of the source distribution is, therefore, larger than that required for thickness alone, and as a result, considerable internal cross flow exists, which may induce serious flow distortion (i.e., leakage) at the airfoil surface in between control points where the boundary conditions are satisfied. Slotted airfoil applications are particularly sensitive to flow leakage. Not only is it possible for flow distortions on one surface to affect the control points on a neighboring one, but, because of the way the Kutta condition is applied[1,2] the overall circulation can be affected. Several recent attempts to minimize leakage effects are reported in Refs. 3-6.

From these experiences, it is hypothesized that the surface singularity model should be applied such that the mean line of the airfoil is a streamline of the *internal* flow. In this way, flow distorions at the surface would be minimized. Such a constraint would clearly require that the magnitudes of both the source and vortex distributions be equal at corresponding points on the upper and lower surfaces. By enforcing this simple symmetry condition, "doublet" effects between the upper and lower surfaces would be eliminated, and the source and vortex singularities would again work in harmony (i.e., as in the linearized theory), each providing its own characteristic to the flow. In such a model, the correct singularity would become dominant in limiting cases where either incidence or thickness goes to zero.

Received April 12, 1976; revision received June 11, 1976.
Index categories: Aircraft Aerodynamics (including Component Aerodynamics); Subsonic and Transonic Flow.
*Senior Research Scientist.
†Vice President. Member AIAA.

Two-Dimensional Method

A simple two-dimensional method was constructed using the symmetrical model. The airfoil surface is represented by an inscribed polygon with an *even* number of sides or panels. Each panel has a constant source distribution and a linearly varying vorticity distribution. The even number of panels allows corresponding upper and lower surface panels to be defined such that they have equal singularity strengths. If there are N panels, then there are $N/2$ unknown source strengths. The vorticity unknowns are located at the panel edges, and that would have given an additional $N/2+1$ unknown values; however, one of these is eliminated by applying the Kutta condition of zero load at the trailing edge. This simply requires the vorticity value there to be zero. The *total* number of unknowns, therefore, is N, and we obtain N simultaneous linear equations by applying the boundary condition of zero normal velocity at the center point (the control point) of each panel. The solution gives the (symmetrical) source and vorticity distributions directly, and the pressures can then be evaluated at the control points.

Results

The method was first applied to the symmetrical Karman-Trefftz section considered in Fig. 1. The same 54 panels were used as in the previous case, and the resulting pressure distribution is compared with the exact solution for the region near the trailing edge in Fig. 2. Also shown are the results from the base method, which is similar to the method of Ref. 2. This new solution is seen essentially to eliminate the

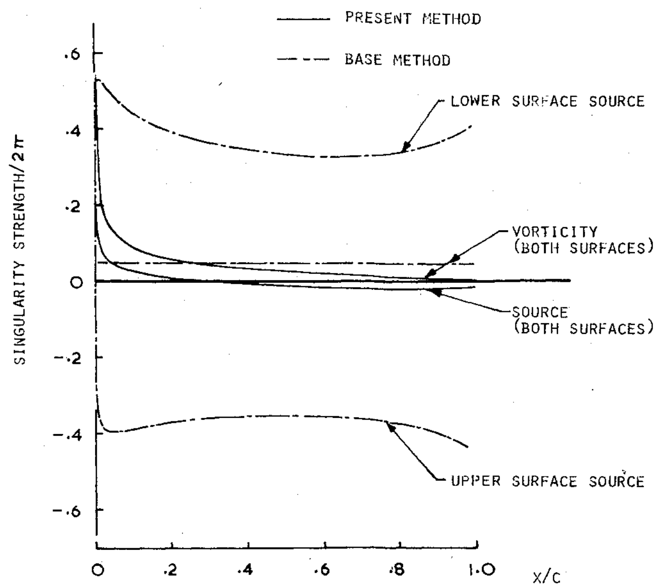

Fig. 1 Surface source and vorticity distributions for a symmetrical Karman-Trefftz airfoil at 10° incidence.

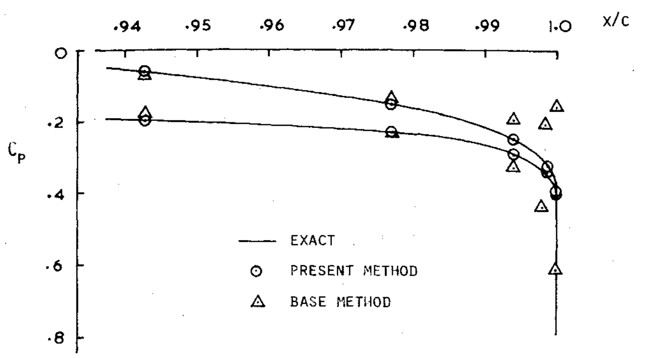

Fig. 2 Detailed pressure distribution in the trailing edge region of the symmetrical Karman-Trefftz airfoil.

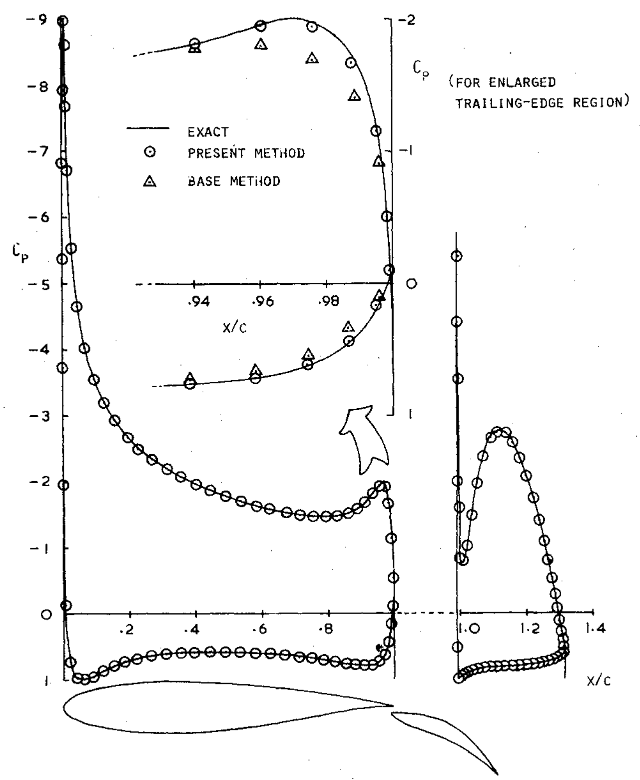

Fig. 3 Pressure distribution on Williams two-component airfoil at zero incidence.

problems previously encountered. Singularity distribution from the solution are well behaved (Fig. 1), and are essentially minimized. They follow the expected trends; the sources being related to rate of change of thickness and the vorticity distribution resembling the incidence loading shape, i.e., the first term of the Birnbaum series $[(1-x)/x]^{1/2}$. It should be emphasized here that , by virtue of the enforced symmetry, the distribution of normal velocity along the chord-line would be zero everywhere; in fact, an examination of surface pressures (not presented here), calculated at points other than control points, has shown that leakage effects have been reduced to a very low order.

For a more severe test case, the method was next applied to Williams' two-component airfoil[7] using a total of 124 panels. In this case, the upper and lower surface panels did not match exactly across the mean line (in practice, the matching could be achieved automatically within the computer program). Even so, the resulting pressure distributions on the two components are in excellent visual agreement with the exact solution (Fig. 3) and show a significant improvement over the base method in the trailing edge region of the leading component. The calculated lift coefficients (based on circulation) for the main airfoil and flap are 2.7801 and 0.9567 respectively compared with the exact values 2.7818 and 0.9568. The corresponding vorticity and source distributions (not presented here) are, again, well behaved. It is interesting that if these vorticity model in the base method, then the problems mentioned in the introduction are virtually eliminated. In general, however, the correct weighting function is not known a priori, particularly in three-dimensional flow.

Conclusions

A method based on symmetrical distributions of sources and vorticity offers an attractive approach to the solution of general potential flow problems. It offers a number of advantages over the widely used surface source/internal doublet or/surface vorticity methods.

1)The Kutta condition is satisfied automatically because the loading goes to zero at the trailing edge. There is no problem over choice of the "Kutta point."

2)Internal flow across the mean line is eliminated for symmetrical sections; the same should be approximately true for cambered sections. The corresponding surface flow distortions in between the control points are thereby minimized without increasing the number of unknowns, and without increasing the computing effort.

3)The singularity strength values are automatically minimized without resorting to special procedures; violently opposing singularity strengths on upper and lower surfaces are eliminated. Such features should be of particular benefit to the convergence of iterative solution techniques. Further, the regular behavior of the singularity distributions would be beneficial in a design version of this method.

4)No arbitrary weighting functions are required by the method. The correct loading form for the vorticity distribution is part of the solution.

All four factors are particularly favorable in applications to multiple component problems.

Although the method used here is based on piecewise constant sources and piecewise linear vorticity, the same symmetrical principle could also be applied to higher order singularity distributions. However, in view of the accuracy achieved by this simple approach using planar panels, development to higher order distributions with curved panels might appear unnecessary except, possibly to reduce the number of unknowns.

References

[1]Rubbert, P.E., Saaris, G.R., Scholey, M.B., and Standen, N.M., "A General Method for Determining the Aerodynamic Characteristics of Fan-Wing Configurations, Vol. I: Theory and Applications," USAAVLABS TR-67-61A, Dec. 1967.

[2]Hess, J.L.,"Calculation of Potential Flow about Arbitrary Three-Dimensional Lifting Bodies," McDonnell Douglas Corp., Long Beach, Calif., MDC J5679-01, Oct. 1972.

[3]Hess, J.L., "The Use of Higher-Order Surface Singularity Distributions to Obtain Improved Potential Flow Solutions for Two-Dimensional Lifting Airfoils," *Computer Methods in Applied Mechanics and Engineering 5*, North Holland Publishing Co., Amsterdam, 1975, pp. 11-35.

[4]Johnson, F.T. and Rubbert, P.E., "Advanced Panel-Type Influence Coefficient Methods Applied to Subsonic Flows," AIAA Paper 75-50, Pasadena, Calif. 1975.

[5]Bristow, D.R., "A New Surface Singularity Method for Multi-Element Airfoil Analysis and Design," *AIAA Journal*, to be published.

[6]Maskew, B., "A Submerged Singularity Method for Calculating Potential Flow Velocities at Arbitrary Near-Field Points," NASA TM X-73, 115, March 1976.

[7]Williams, B.R., "A Comparison of the Surface Source Solution with an Exact Solution for the Two-Dimensional Inviscid Flow about a Slotted-Flap Aerofoil," Royal Aircraft Establishment, Farnborough, England, TR 72008, March 1972.